Quantum interference in electron collision

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The indistinguishability of identical quantum particles can lead to quantum interferences that profoundly affect their scattering1,2. If two particles collide and scatter, the process that results in the detection of the first particle in one direction and the second particle in another direction interferes quantum mechanically with the physically indistinguishable process where the roles of the particles are reversed. For bosons such as photons, a constructive interference between probability amplitudes can enhance the probability, relative to classical expectations, that both are detected in the same direction—this is known as ‘bunching’. But for fermions such as electrons, a destructive interference should suppress this probability (‘anti-bunching’); this interference is the origin of the Pauli exclusion principle, which states that two electrons cannot occupy the same state. Although two-particle interferences have been shown for colliding photons3,4, no similar demonstration for electrons exists5,6. Here we report the realization of this destructive quantum interference in the collision of electrons at a beam splitter. In our experiments, the quantum interference responsible for the Pauli exclusion principle is manifest as the suppression in electron current noise after collision.

The scattering of individual particles at a beam splitter is a stochastic process that introduces fluctuations (partition noise) in the output flux, depending on the transmission probability, $T$ (Fig. 1a). If $N$ identical particles are independently scattered in series, a binomial distribution results for the number of particles transmitted to one of the outputs, $N_{\text{out}}$, and the normalized variance (Fano factor) is equal to $(\Delta N_{\text{out}}^2/N_{\text{out}}) = (1 - T)$. In the limit of an extremely small $T$, the particle noise approaches the Poisson limit (full shot noise). This single-particle partition process is independent of the quantum statistics of incident particles.

This no longer holds in the case of collisions (Fig. 1b and c). Assuming a beam splitter with $T = 1/2$, the collision of classical particles (independent partitioning of the two inputs) results in probabilities of 1/4 for two particles to be scattered to the left or two to the right, whereas the probability that one particle is scattered into each output is 1/2. If the incident particle fluxes do not fluctuate, each output flux exhibits one-half shot noise.

In the quantum mechanical case, because particles are described by probability amplitudes, when the wavefunctions of two identical particles initially on the left and right overlap, it becomes impossible to distinguish one particle from the other. Applying the postulates of quantum mechanics can then in principle yield results that depend on whether the state is described mathematically by $|1: \psi_1; 2: \psi_2\rangle$ (particle 1 on the left and particle 2 on the right) or by $|1: \psi_2; 2: \psi_1\rangle$ (vice versa). Only the symmetric and

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antisymmetric combinations of these states, $|\psi_{-}\rangle = (1/\sqrt{2})[|1:\psi_1;2:\psi_2\rangle \pm |2:\psi_1;1:\psi_2\rangle]$, produce real measurement results. Particles having overall symmetric wavefunctions are called bosons, and those having antisymmetric wavefunctions are called fermions. The photon is a classical example of a boson, whereas the electron is a classic example of a fermion.

The probability amplitudes for the three possible collision outcomes are derived from the evolution of the two-particle wavefunction under the influence of the beam splitter scattering matrix (unitary evolution). The symmetrization or antisymmetrization of the wavefunction yields two contributions to the probability amplitude, a direct and an exchange term. If we consider the output state where both particles are scattered to the left, then both of these terms have the same magnitude and sign (Fig. 1d). For bosons, these terms add to give a probability of $1/2$. For fermions, these terms subtract to give completely destructive interference; two fermions never scatter into the same state, a manifestation of the Pauli exclusion principle.

If we consider the output state where one particle is in each output, then, because the unitary evolution introduces a minus sign for reflection from the right input to the right output in order to satisfy power conservation, the direct and exchange terms have the same magnitude but opposite signs (Fig. 1e). Completely destructive interference is now obtained for bosons, whereas the probability for fermions equals one. In this ideal beam splitter, whereas two bosons always scatter into the same port (Fig. 1b) so that the fluctuations in the output flux exhibit full shot noise (although not poissonian noise), one fermion always scatters into each port (Fig. 1c) so that the fluctuations in the output flux are completely suppressed. The fact that quantum interferences profoundly affect the collision noise suggests that a measurement of the output noise should confirm the quantum statistics of the particles.

To observe this, we fabricated a mesoscopic electron beam splitter by electron beam lithography on a GaAs high-mobility, two-dimensional electron gas system (Fig. 2a). The two input and two output ports are defined by point contact constrictions formed by an etched trench and Schottky gates that deplete underlying electrons when negative gate biases are applied. A 40-nm gate finger down the centre of the scattering region allows the splitting ratio of the beam splitter to be tuned over a narrow range around 50:50 partitioning of transmitted electrons. The transport through the device is primarily ballistic, as the length scales for inelastic phonon scattering and elastic ionized impurity scattering at an operating temperature of 1.6 K are much longer than the device size.

To improve the signal-to-noise ratio in the noise measurement, an a.c. modulation scheme$^{37}$ is used with a cryogenic cascode configuration preamplifier.

The use of ballistic point contacts for the inputs is necessary to achieve streams of single-mode electron wavepackets incident on

Figure 2 Suppression of collision noise in an electron beam splitter. a, Scanning electron micrograph of an electron beam splitter device fabricated on a GaAs two-dimensional electron gas system. Schottky gates and an etched trench (dark area near the centre gate) define the inputs (gate–gate point contacts) and the outputs (gate–trench point contacts). b, The magnitude of the current noise power for the right output (arbitrary units), as a function of its current. A modulation scheme and resonant circuit are used to improve the discrimination between the stationary background amplifier noise and the current-dependent partition noise, and thus improve the signal-to-noise ratio at the $15.6\times 10^6$ measurement frequency. When either the left input (downward triangles) or the right input (upward triangles) is biased individually, the single-particle partition noises should be and are roughly the same, as the transmission probabilities into the right output are similar. Their slopes are used as references to compare the noise during collision (squares), which is clearly suppressed. c, After normalizing the noise by the current (after subtracting zero bias offsets) and scaling by the classical collision noise determined from the weighted average of the slopes of the single-particle partition noises, the measured collision noise can be compared to the classical limit. Its suppression indicates a fermion interference. The noise is not completely suppressed, in part because of non-idealities in the beam splitter's scattering matrix.
the beam splitter. Both the conductance and noise properties of such a device have been actively studied. For our purposes, measurements on an isolated point contact (Fig. 3 inset) serve to illustrate the expected noise contribution of the Schottky gate point contacts used as inputs in the collision experiment. The conductance through the point contact is proportional to the transmission probability for electrons at the Fermi energy. As the width of a point contact increases to half the de Broglie wavelength of the electrons (typically of the order of 100 nm), partial transmission through its lowest transverse mode becomes possible, and the conductance increases with decreasing partition noise. Once the lowest transverse mode in the point contact is fully transmitting, a plateau is reached in the conductance corresponding to the quantum unit of conductance (with spin degeneracy), $G_0 = 2e^2/h$, and the partition noise is completely suppressed (Fig. 3). Electrons are now steadily injected into the single mode of the point contact and are transmitted without stochasticity. Then, as the width of the point contact increases further, the higher transverse modes open up one by one with partition noise associated only with the opening mode. This is suggested by the fact that the noise normalized by the value of full shot noise corresponding to the total current shows peaks whose magnitude decreases as the transmission of the highest opening mode passes through 1/2.

In the electron collision experiment, similar Schottky gate point contacts form the two inputs of the beam splitter, and both are set at their first conductance plateau where electrons are steadily injected with unity transmission. However, because of device non-idealities leading to finite reflection from the beam splitter, the input conductances at this plateau are less than $G_0$, and the overall transmissions into the right output (where the noise is measured) are only about 35% (left input) and 30% (right input). The reflections result in additional partition noise and cause the collision noise power to increase with current. Complete noise suppression (as in the ideal model) is no longer possible because after collision, the electrons will occasionally leave through the input ports, causing the electron flux in the right output to fluctuate. These fluctuations, however, should still be smaller than the purely classical case, owing to the quantum interference.

The output noise power is plotted in Fig. 2b against the output current when each input port is biased individually and when both input ports are simultaneously biased. The linear current dependence is a sign of noise due to partitioning. Because the transmissions into the right output are about the same for the two inputs, the slopes for the individual bias cases should be and are roughly the same. On the other hand, it is evident that the slope in the case of collision is smaller. Figure 2c emphasizes the significance of this by reploting the data as the normalized current noise relative to the expected classical collision noise determined from the sum of the individual partition noise powers. Given the transmission probabilities and using a coherent scattering formalism assuming a model for the device that accounts only for the overall transmissions from inputs to outputs and disregards path length differences, the fermionic collision noise should be 52% of the classical collision noise. The observed suppression of the measured collision noise to an average of 56% of the classical value therefore indicates the presence of the fermion quantum interference.

The materials and methods used in this demonstration of fermion interference should be useful for studying other fluctuation phenomena in highly degenerate electron systems where quantum statistical effects can be manifest in the second-order coherence function. Further experiments on the quantum mechanics of fermions, such as studying intensity correlations from a thermal electron source, analogous to the Hanbury Brown and Twiss experiment for photons, should be possible in this new field of quantum electron optics.

Figure 3 Conductance and partition noise in a quantum point contact. Inset, scanning electron micrograph of an isolated quantum point contact device. Main figure, conductance (solid, red) and current noise normalized by the full shot noise value (dotted) (scaled to the expected Fano factor of 1/2 at $T = 1/2$ in the first plateau), as a function of the voltage applied to the confining Schottky gates. The current through a point contact is carried by discrete transverse modes. As the point contact width is increased with decreasing negative gate bias, the transmission probability of a mode increases from zero to unity as its transverse sub-band energy drops below the Fermi energy. Once fully transmitting ($T = 1$), a transverse mode carries a fixed current per unit voltage corresponding to $G_0$, and a plateau is reached in the conductance. The partition noise is absent at these plateaux. The current noise is measured for four different source-drain bias voltages of 0.5 (green), 0.75 (brown), 1.25 (blue) and 1.75 mV (violet). The noise power peaks at the middle of the steps where $T = 1/2$ for the newly opening mode. The theoretical behaviour of the partition noise, based on the measured conductance assuming all fully opened modes have $T = 1$, is also plotted (solid, black) and shows that the trend in experimental noise peaks agrees with theoretical prediction.